# Order Recursive Method of Moments : A Powerful Computational Tool for Microwave CAD and Optimization

Krishna Naishadham and Pradeep Misra

Department of Electrical Engineering
Wright State University, Dayton, OH 45435

**Abstract:** The method of moments (MoM) continues to be the most frequently used electromagnetic simulation technique for application to CAD and optimization of microwave circuits. In this paper, we present an order-recursive variant of standard LU decomposition for the efficient solution of large linear systems arising in the application of MoM to iterative design problems. In comparison with the existing matrix methods, which solve the linear sytem from scratch at each design iteration, the proposed order-recursive MoM allows a very large portion of recomputation to be avoided, leading to extremely efficient design iterations. Therefore, this contribution is a major advance toward EM simulation-based (or full-wave) CAD and optimization of microwave circuits.

## 1. Introduction

Increasing operating frequencies in digital and microwave systems have necessitated the utilization of full-wave electromagnetic (EM) simulation techniques, such as the method of moments (MoM) [1], the finite element method [2], and the finite-difference time-domain (FDTD) method [3], for the analysis of high-speed digital circuits, microwave and millimeter-wave integrated circuits (MMICs). These rigorous techniques account for physical phenomena such as surface-wave coupling and radiation, dispersion, metallization and dielectric losses. However, they are typically computation-intensive, and therefore, are of limited use in the computer-aided design (CAD) and optimization of microwave circuits. Therefore, it is desirable to investigate means of improving the efficiency of full-wave methods so that a circuit of moderate electrical size can be simulated in reasonable time on a personal computer (PC) or a workstation. These computational platforms, and not large-scale mainframes or supercomputers, seem to represent the typical design environment in the microwave industry.

The MoM is perhaps the most widely used EM simulation method for the analysis and CAD of microwave circuits. Several software houses offer commercial MoM-based simulation tools for the microwave circuit designer (*e.g.*, [4] - [9]). These software packages have the capability to incorporate optimization features either internally [5], or by coupling the MoM simulation engine to an external parametrized optimizer [4], [9]. Several researchers are also investigating methods to improve the efficiency of MoM-based circuit simulators so that robust CAD tools and optimizers can be developed for smaller computational platforms. These investigations include the application of wavelet transforms to reduce the order of the moment matrix [10], utilization of symmetries and redundancies in the problem space to efficiently fill the matrix[11], space mapping optimization techniques [12], and order-recursive linear system solvers [13].

In the MoM, the boundary value problem for the unknown current distribution over the surface of the conductors is formulated as an electrical field integral equation (EFIE). The EFIE is then converted into a system of linear algebraic equations (for the current) by the application of suitable basis and testing functions. Parameters of interest, such as $S$-parameters, radiation and metallization losses, can be derived from the computed current distribution. The current distribution can be computed by implementing the MoM algorithm either in the space domain (*cf.* [14]) or in the spectral domain (*cf.* [15]).

The system (or moment) matrix that represents the interactions between the basis and the test elements is typically dense. For moderately high-order models ($\mathcal{O}(100-500)$), the current distribution may be obtained as the solution of a system of linear algebraic equations using LU decomposition and subsequent solution of two triangular systems of equations. The computational complexity of the solution of system of equations of order $N$ is $N^3$. For several applications, where $N$ is fixed, the use of conventional LU decomposition provides an efficient means for solving the linear systems.

However, in design applications, the order of the linear system to be solved may change from $N$ to $N+M$, where the original $(N \times N)$ data matrix becomes a submatrix of the higher-order $(N+M) \times (N+M)$ matrix as a result of augmenting the model. This is frequently

TH 2E



0-7803-3246-6/96/$5.00 © IEEE

1996 IEEE MTT-S Digest

encountered, for example, in the tuning of patch antennas and microwave filters, where the data matrix is recursively augmented with new row and column vectors that correspond to shorting pins, stubs, etc. In a CAD environment, the order $M$ of augmentation is usually not known *a priori.* At present, each augmented matrix is treated as a new data matrix and the solution of the augmented system of equations is recomputed from scratch. The resulting solution procedure becomes computationally inefficient, and, as shown elsewhere [13], the computational complexity can become $\mathcal{O}((N+M)^4)$. The authors' previous investigations reveal that when small changes are made to the circuit geometry, they correspond to changes in a small subsystem of the original linear system to be solved for the current distribution. The existing software packages with EM simulators based on the moment method do not take advantage of the fact that most of the computations performed at a previous iteration can be embedded into the new system of algebraic equations. Instead, they remodel the entire system and solve the system so obtained from scratch. The objective of this paper is to apply an order-recursive variant of LU decomposition to develop a solution procedure which will allow a very large portion of recomputation to be avoided, leading to extremely efficient design iterations. Therefore, this contribution is a major advance toward EM simulation-based (or full-wave) CAD and optimization of microwave circuits. The computational complexity of the proposed method, termed as **order recursive method of moments (ORMoM)**, is $\mathcal{O}((N+M)^3)$. Clearly, this order of magnitude reduction in computations is very attractive for interactive design tasks.

The computational efficiency of ORMoM is illustrated by applying it to two design problems. First, we consider the tuning of stub-loaded dual-band patch antennas [16], which find application in mobile communications systems because of their light weight, low profile and ease of fabrication. It is well-known that rectangular microstrip patch antennas with reactive stubs placed along the radiating edges exhibit dual frequency operation [16], [17]. Next, we iteratively "fine-tune" the design of a folded double-stub microstrip filter to achieve the desired specifications in the pass-band for the insertion loss. At each iteration, the spacing between, or the length of the stubs, is varied in either direction (incremented or decremented), as would be typically required in a circuit optimization environment, and, the modified system of equations is solved efficiently for the current distribution by using order recursion.

## 2. Order Recursive LU Decomposition

The proposed algorithm assumes that *all* the leading principal submatrices of the system (or moment) matrix $\mathbf{A}$ are nonsingular. Therefore, the solution (albeit suboptimal) can always be computed without the need of pivoting. Assume that the LU decomposition of $A$ (=$LU$) has been computed. Denote an augmented matrix and its LU decomposition as:

$$A := \left[\begin{array}{c|c} A_{11} & \mathbf{a}_{12} \\ \hline \mathbf{a}_{21} & a_{22} \end{array}\right] = \left[\begin{array}{c|c} L_{11} & \mathbf{o} \\ \hline \mathbf{l}_{21} & l_{22} \end{array}\right] \left[\begin{array}{c|c} U_{11} & \mathbf{u}_{12} \\ \hline \mathbf{o} & 1 \end{array}\right]. \tag{1}$$

Then, it is readily seen that the unknowns may be computed as:

$$\begin{aligned} \mathbf{u}_{12} &= L_{11}^{-1}\mathbf{a}_{12} \\ \mathbf{l}_{21} &= \mathbf{a}_{21}U_{22}^{-1} \\ l_{22} &= a_{22} - \mathbf{l}_{21}\mathbf{u}_{12}. \end{aligned} \tag{2}$$

Of course one *must not* compute the inverses of matrices as shown in (2). Instead, the unknowns are obtained by solution of the triangular system of equations, which is known to be numerically stable. The above result is easily extended to the case when the matrix is bordered by several rows and columns. Specifically, if

$$A := \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right] = \left[\begin{array}{c|c} L_{11} & O \\ \hline L_{21} & L_{22} \end{array}\right] \left[\begin{array}{c|c} U_{11} & U_{12} \\ \hline O & U_{22} \end{array}\right], \tag{3}$$

then, the unknowns may be computed as:

$$\begin{aligned} U_{12} &= L_{11}^{-1}A_{12} \\ L_{21} &= A_{21}U_{11}^{-1} \\ L_{22}U_{22} &:= A_{22} - L_{21}U_{12} \end{aligned} \tag{4}$$

where, the last equation in (4) represents an LU decomposition of the matrix on its right-hand side. We have developed a similar approach for solving **decremented** moment systems, which pertains to the *removal* of certain elements or metallization in a structure. This situation is encountered, for example, when the stub length or separation in a microstrip filter needs to be decreased iteratively. In this case, the order of the new matrix is smaller than that of the original matrix. For brevity, we have not presented the solution procedure pertaining to decremented systems.

## 3. Design Examples

The solution procedure described in Section 2 is applied to the iterative design of a coax-fed tunable dual-band patch antenna, whose geometry is shown in Fig. 1. The substrate is 0.79 mm thick duroid ($\epsilon_r$ =2.17). The current distribution on the structure is computed using an efficient PC-based MoM simulator described in [11]. The patch and the stub are gridded into a rectangular mesh which supports rooftop basis functions. The resulting moment matrix for the patch alone is of the order 216. The return loss of the untuned patch is shown in Fig. 2 and a fundamental resonance is observed at 2.5 GHz. In order to provide dual-band operation, an open-circuited $\lambda/4$ monolithic stub is connected perpendicular to a radiating edge (Fig. 1) and its position along the edge or its length is varied iteratively. At each iteration,

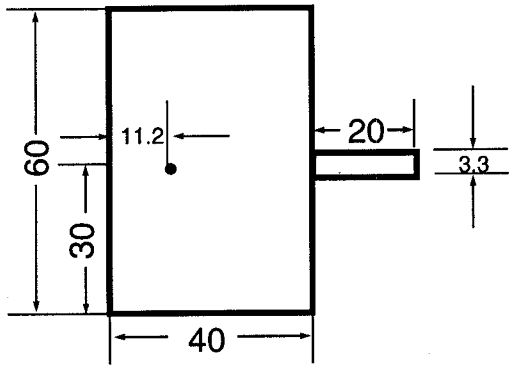


**Figure 1:** Patch antenna geometry.

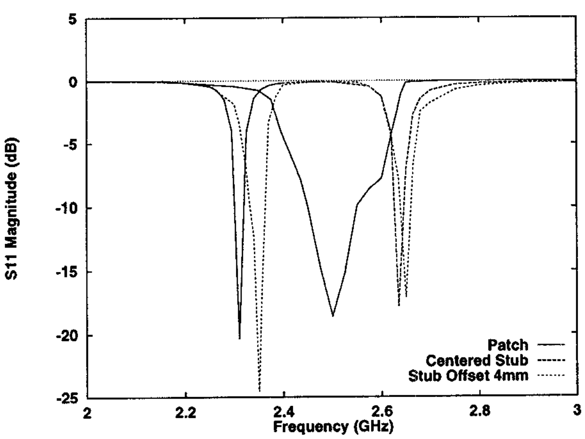


**Figure 2:** Return loss of the tuned patch antenna.

instead of solving the linear system of moment equations from scratch, the LU decomposition of the patch is efficiently utilized in solving the augmented system created by the addition of the stub.

Two positions of the 20 mm long stub are simulated: (a) center of the radiating edge, (b) 4 mm above the center. In either case, the 216 × 216 system matrix of the patch is augmented by 8 rows and columns. Location (a) produces two resonances at 2.31 and 2.635 GHz (Fig. 2) in contrast to the experimentally observed values, 2.275 and 2.666 GHz, respectively [16]. The separation of the resonances can be varied by moving the stub along the radiating edge. Location (b) produces the resonances at 2.35 and 2.65 GHz (Fig. 2), clearly demonstrating the tuning nature of the stub. Because of symmetry, a similar band separation has been observed when the stub is located 4 mm below the center of the radiating edge.

As an indication of computational efficiency of ORMoM, the conventional MoM implementation of solving the currents from scratch at each iteration would have required $216^3 + 224^3 = 21,317,120$ operations, whereas ORMoM requires only $216^3 + 2 \times 8 \times 216^2 = 10,824,192$ operations (savings of about 50%). It is evident that the computational savings would be larger for higher-order problems, and for those involving many iterations (*e.g.*, in optimization).

Next, we consider the design of a folded double-stub microstrip filter seen in Fig. 3 with design specifications given by [12]:

$$|S_{21}| \geq -3\text{dB}, \quad \begin{cases} f \leq 9.5\text{GHz} \\ f \geq 16.5\text{GHz} \end{cases}$$

$$|S_{21}| \leq -30\text{dB}, \quad \begin{cases} f \leq 14\text{GHz} \\ f \geq 12\text{GHz} \end{cases}$$

The substrate has a thickness of 5 mils and $\epsilon_r = 9.9$. The two stubs have the same width as the main line, given by $W_1 = W_2 = 4.8$ mils, and equal length. The parameters $L_1$, $L_2$ and $S$ are varied iteratively as shown in Table 1, and the performance of the filter is evaluated in terms of the insertion loss at each iteration until the design specifications are met. Note that changing $L_1$ alters the horizontal spacing between the stubs, whereas changing $L_2$ varies the length of the stubs. Variation of $S$ affects the vertical spacing between the stubs. The three designable parameters are either incremented or decremented at each iteration following the entries in Table 1, and ORMoM is employed to efficiently compute the current distribution.

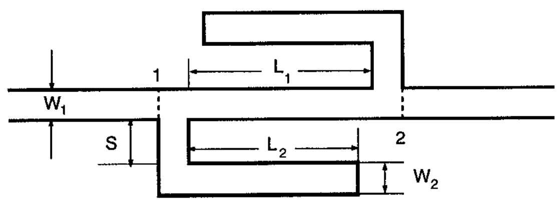


**Figure 3:** Microstrip double folded-stub filter.

The insertion loss (magnitude of $S_{21}$ in dB) and return loss of the filter are plotted in Figs. 4 and 5, respectively. The design in first iteration does not meet the specifications on insertion loss at several frequencies in the 12 to 14 GHz band. Also, both of the notches are quite discrepant from the optimized solutions of approximately 12 GHz and 14 GHz [12]. The subsequent iterations are observed to improve the insertion loss toward the specifications, with the fourth iteration yielding a response close to the best-optimized fine model simulation in Fig. 9 of [12]. In a frequency band of approximately 0.5 GHz around 13 GHz, the insertion loss for all the iterations is offset from the design specification of -30 dB by about 2 dB. This is perhaps within the numerical accuracy of the moment method implementation used in either investigation. The return loss shown in Fig. 5 is nominally less than -10 dB in each passband around the central notch of 13 GHz, and clearly demonstrates the band-reject nature of the filter.

| Iter. | $L_1$ | $L_2$ | $S$ |
|---|---|---|---|
| 1 | 90 | 80 | 4.8 |
| 2 | 91.5 | 85.7 | 4.1 |
| 3 | 93.7 | 85.3 | 4.6 |
| 4 | 92.1 | 85.1 | 4.2 |

**Table 1:** Parameters for iterative design of the folded-stub filter

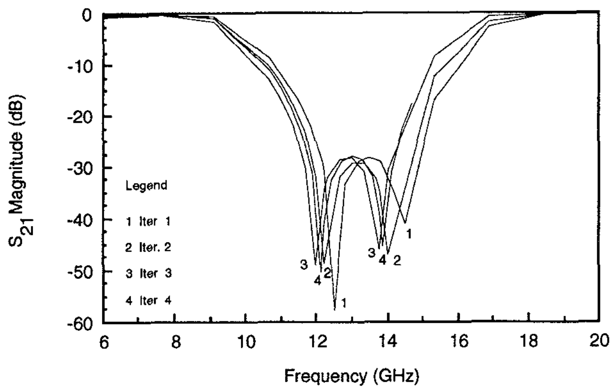


**Figure 4:** Iterative double folded-stub filter design: $|S_{21}|$.

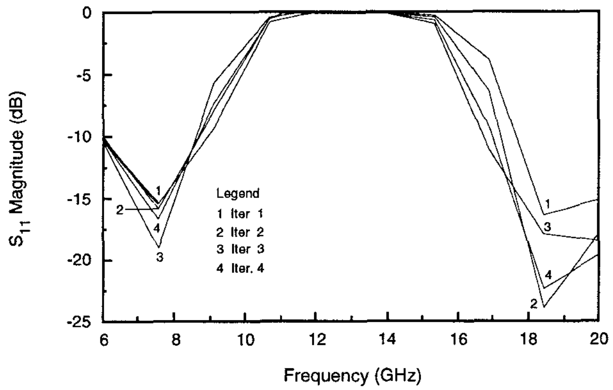


**Figure 5:** Iterative double folded-stub filter design: $|S_{11}|$.

## 4. Conclusions

An order-recursive variant of conventional LU decomposition has been presented for the efficient solution of linear systems arising in an iterative moment method simulation, with potential application to microwave CAD and optimization. The method is illustrated by interactive design of a tunable dual-band microstrip patch antenna and a microstrip band-reject filter. We are currently investigating the application of ORMOM to improve solution accuracy and numerical resolution of the simulation by increasing the cell density in regions of high field variation, keeping lower density in others. The goal is to provide an efficient full-wave analysis tool for EM optimization, which does not have the serious limitation [12] of having the same cell size in all computational regions.

## References

[1] R.F. Harrington, *Field Computation by Moment Methods*, New York, NY: Macmillan, 1968.

[2] J. Jin, *The Finite Element Method in Electromagnetics*, New York, NY: John Wiley, 1992.

[3] A. Taflove, *Computational Electrodynamics: The Finite-Difference Time-Domain Method*, Norwood, MA: Artech House, 1995.

[4] *em*, Sonnet Software, Liverpool, NY 13090.

[5] *IE3D*, Zeland Software, Inc., Fremont, CA 94538.

[6] *Ensemble*, Boulder Microwave Technologies, Inc., Boulder, CO 80302.

[7] *Explorer*, Compact Software, Paterson, NJ 07504.

[8] *Momentum EM Simulator*, Hewlett Packard, Westlake Village, CA 91362.

[9] *OSA90/hope* and *EMpipe*, Optimization Systems Associates, Inc., Dundas, Ontario, Canada.

[10] K. Sabetfakhri and L.P.B. Katehi, "Analysis of integrated millimeter-wave and submillimeter-wave waveguides using orthonormal wavelet expansions," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-42, pp. 2412-2422, December 1994.

[11] K. Naishadham and T.W. Nuteson, "Efficient analysis of passive microstrip elements in MMICs", *Int. J. Microwave and Millimeter-Wave Comp. Aided Eng.*, vol. 4, no. 3, pp. 219–229, 1994.

[12] J. Bandler et al., "Space mapping technique for electromagnetic optimization", *IEEE Trans. Microwave Theory Tech.*, vol. MTT-42, pp. 2536-2544, December 1994.

[13] K. Naishadham and P. Misra, "Order recursive Gaussian elimination and efficient CAD of microwave circuits," *IEEE Microwave Symp. Dig.*, pp. 1435-1438, May 1995.

[14] J.R. Mosig, "Integral Equation Technique," in *Numerical Techniques for Microwave and Millimeter-Wave Passive Structures*, (T. Itoh, ed.), New York, NY: John Wiley, pp. 133-213, 1989.

[15] R. H. Jansen, "The spectral-domain approach for microwave integral circuits," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 1043-1056, October 1985.

[16] S.E. Davidson, S.A. Long, and W.F. Richards, "Dual band microstrip antenna with monolithic reactive loading," *Electron. Lett.*, vol. 26, pp. 936-937, 1985.

[17] J.R. James and P.H. Hall, *Handbook of Microstrip Antennas*, vol. I, Peter Peregrinus, London, 1989.